3.3 Derivatives 2014-10-13 day 33 COLUMBUS DAY

3.2/32 $y=\frac{x^2}{4}$; tangent at $x=1$; secant $(0,0)\to(2,1)$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{(x+h)^2}{4}-\frac{x^2}{4}}{h}$$

$$=\lim_{h\to 0}\frac{\frac{x^2+2xh+h^2-x^2}{4}}{h}=\lim_{h\to 0}\frac{2xh+h^2}{4h}$$

$$=\lim_{h\to 0}\frac{h(2x+h)}{h(4)}=\lim_{h\to 0}\frac{2x+h}{4}=\frac{2x}{4}=\boxed{\frac{x}{2}}$$

$$f'(x)=\frac{x}{2}$$

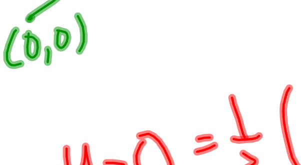
$f'(1)=\frac{1}{2}$ ⇐ slope Point ⇒ $(1,f(1))=\left(1,\frac{1^2}{4}\right)=\left(1,\frac{1}{4}\right)$

eqn of tangent line: $y-\frac{1}{4}=\frac{1}{2}(x-1)$

$y-\frac{1}{4}=\frac{1}{2}x-\frac{1}{2}\Rightarrow y=\frac{1}{2}x-\frac{1}{4}$

b) secant: $\frac{f(\text{end})-f(\text{begin})}{\text{end}-\text{begin}}=\frac{1-0}{2-0}=\boxed{\frac{1}{2}}$

average rate of chg. = slope of secant line



eqn of secant line

[use (0,0)] $y-0=\frac{1}{2}(x-0)\Rightarrow y=\frac{1}{2}x$

[use (2,1)] $y-1=\frac{1}{2}(x-2)\Rightarrow y-1=\frac{1}{2}x-1$

$=y=\frac{1}{2}x$

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$$\frac{\frac{a}{b}\left(\frac{1}{c}\right)}{c\left(\frac{1}{c}\right)} = \frac{\frac{a}{bc}}{1}$$

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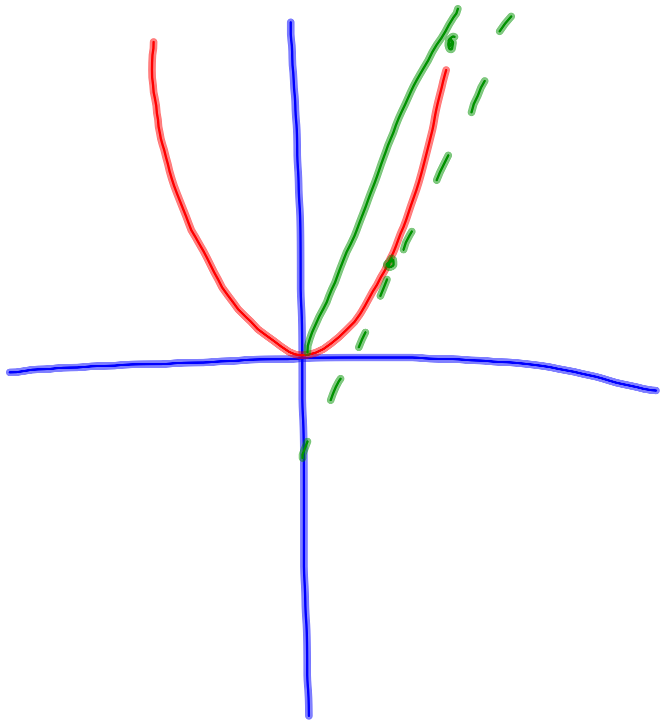

3.3 Derivatives

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$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$\boxed{28a}$

$$\lim_{x \to 1} \frac{x^7 - 1}{x - 1}$$

limit definition of derivative of $f(x) = x^7$ at $x = 1$

suspect:

$a = 1$

$f(a) = 1$

$f(x) = x^7$

consistent

WHY? → stay tuned, grasshopper

3.3 Derivatives

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28b $\displaystyle\lim_{h\to 0} \frac{\cos(\pi+h)+1}{h}$

x+h or a+h

$-(-1)$

$\displaystyle f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

suspect

$a=\pi$

$f(a)=-1$

suspect

$f(x)=\cos(x)$

$\cos(\pi)=-1$?

yes

$\displaystyle f'(\pi)=\lim_{h\to 0}\frac{\cos(\pi+h)\overbrace{-\cos(\pi)}^{\overbrace{-(-1)}^{+1}}}{h}$

when $f(x)=\cos(x)$

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3.3) Rather than use a limit definition to calculate a derivative EVERY SINGLE TIME, mathematicians of days gone by have looked for patterns.

3.3 Derivatives

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$\frac{d}{dx}(c) = 0$

[$\frac{dy}{dx}$ if we know what y is]

$\frac{d}{dx}(x) = 1$

[$\frac{d}{dx}$ to take the derivative of what follows IN THE PARENTHESES]

$\frac{d}{dx}(mx) = m$

based on limit

$\frac{d}{dx}(c\,f(x)) = c \cdot \frac{d}{dx}(f(x))$

$\frac{d}{dx}(f(x)+g(x)) = \frac{d}{dx}(f(x)) + \frac{d}{dx}(g(x))$

$\cancel{x^n - a^n}$ $\quad \frac{1}{x^n} - \frac{1}{a^n} \quad \frac{x^n - a^n}{x^n a^n}$

$\Longrightarrow$ power rule

$\frac{d}{dx}(x^n) = n x^{(n-1)}$

FIRST BIG RULE

[based on binomial theorem]

$\frac{d}{dx}(x^4) = 4x^3$

in $\frac{d}{dx}(x^n)$ → exponent is a constant!

variable is in the base!

$\frac{d}{dx}(x^8) = 8x^7$

$\frac{d}{dx}(x^{121}) = 121x^{120}$ . . .

3.3 Derivatives

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First rule for combining 2 functions
(that's hard)

Product Rule shorthand

$$\frac{d}{dx}\left(f(x)\cdot g(x)\right) = \frac{d}{dx}(f\cdot g)$$

$$= f'g + fg'$$

$$\frac{d}{dx}(x^2\cdot x^3) =$$

$$\frac{d}{dx}(x^2)\cdot x^3 + x^2\cdot\frac{d}{dx}(x^3)$$

$$=(2x^1)x^3 + x^2(3x^2)$$

$$=2x^4+3x^4$$

$$=\boxed{5x^4}$$

$$\frac{d}{dx}(x^2\cdot x^3) =$$

$$\frac{d}{dx}(x^5) =$$

3.3 Derivatives

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$$\frac{d}{dx}\left(5x^3+2x\right)$$

$$= 5\frac{d}{dx}(x^3)+2\frac{d}{dx}(x^1)$$

$1\cdot x^0 = 1\cdot 1 = 1$

$$= 5(3x^2)+2(1) = \underline{15x^2+2}$$

$$\frac{d}{dx}\left(15x^2+2\right) = 15\frac{d}{dx}(x^2)+\frac{d}{dx}(2)$$

$$= 15(2x)+0 = 30x$$

$\frac{dy}{dx} = y$

these equations involving derivatives are called differential equations and are EPIC important, dude.

3.3 Derivatives

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Coming up - - - - - -

Quotient Rule

Chain Rule

derivatives of ....

exponential f^n,

logarithmic f^n,

trig f^n,

inverse trig f^{ns}..